Statistical Evaluation of Voting Rules

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This version: February 27, 2015

Accepted for publication in *Social Choice and Welfare* on May 24, 2015

Abstract: We generate synthetic elections using two sources of survey data, two spatial models, and two standard models from the voting literature, IAC and IC. For each election that we generate, we test whether each of 54 voting rules is (1) non-manipulable, and (2) efficient in the sense of maximizing summed utilities. We find that Hare and Condorcet-Hare are the most strategy-resistant non-dictatorial rules. Most rules have very similar efficiency scores, apart from a few poor performers such as random dictator, plurality and anti-plurality. Our results are highly robust across data-generating processes. In addition to presenting our numerical results, we explore analytically the effects of adding a Condorcet provision to a base rule and show that, for all but a few base rules, this modification cannot introduce a possibility of manipulation where none existed before. Our analysis provides support for the Condorcet-Hare rule, which has not been prominent in the literature.

Journal of Economic Literature Classification Codes: D71, D72

Keywords: Voting rules, efficient decision-making, strategy-resistance, manipulability, Condorcet, Hare.

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[*] Corresponding author. Phone: (845) 200-5467. We are grateful to Florenz Plassmann for supplying a copy of the German Politbarometer candidate data that he downloaded and organized. We are also grateful to those who commented on earlier drafts of this paper at the 2013 Public Choice Society conference and the 2013 Southern Economic Association conference.

1. Introduction

When the members of a collective body will be taking a single vote over more than two options, to elect a single officer such as a president or mayor, what voting rule should be used to aggregate the votes and identify the winner? To answer this question we should have a means of identifying the strengths and weaknesses of different single-winner voting rules. One traditional approach is to list logical properties that each rule does or does not possess.[1] This is valuable, but it leaves unanswered such questions as the relative importance of different criteria and the frequency with which any particular failure will cause practical problems in real elections. Therefore some authors have sought to evaluate voting rules in a statistical way, assigning them numerical scores according to their performance in different dimensions, such as resistance to strategic manipulation and utilitarian efficiency.[2] In this paper, we pursue the statistical approach, assessing voting rules' performance in these two dimensions.

To be specific, we use simulated three-candidate elections to evaluate 54 voting rules in terms of resistance to strategy and utilitarian efficiency. These voting rules are divided into eight categories: positional, elimination, Condorcet-positional, Condorcet-elimination, cardinal, Condorcet-cardinal, other comparison-based, and dictatorial. We perform this analysis with each of six data-generating processes, including two based on survey data, two spatial models, and two 'culture' models.

The paper is organized as follows. We motivate our resistance to strategy and utilitarian efficiency statistics in section 2, review the relevant literature in section 3, and provide more detail about how we compute our statistics in section 4. In section 5, we define the voting rules under examination. In section 6, we develop logical propositions that hold regardless of the process used to generate voter preference data, which focus on the effects of modifying a voting rule by adding a provision to elect the Condorcet winner when one exists. In section 7, we describe the data-generating processes that we use in our simulations. In section 8 we present our results from the Politbarometer survey, and in section 9 we compare these with the results from the other five data-generating processes. In section 10, we consider the tradeoff between efficiency and strategic resistance in light of the simulation results, and in section 11 we make concluding remarks. In the appendix, we provide proofs of our propositions.

2. Motivation

Here we establish that resistance to strategy and utilitarian efficiency are each independently valuable as criteria, and furthermore that they are complementary, so that there is an added value in considering them simultaneously when one is deciding which voting rule to adopt.

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[1] For summaries see Richelson (1978), Tideman (2006), and Felsenthal (2012).

[2] Other dimensions investigated include the frequency of electing the Condorcet winner, the frequency of avoiding the election of the Condorcet loser, the frequency of avoiding situations where monotonicity or participation failures might arise, etc. Some voting rules perform perfectly in some of these dimensions, but no voting rule performs perfectly in all of them.

2.1. Resistance to strategy

In this paper we define resistance to strategy operationally as the likelihood that sincere voting will result in an outcome that no group of voters will be able to change to their mutual advantage by changing their votes. This is a valuable property for a voting rule to possess because strategic voting can undermine the political process in several ways. To see this, first consider the plurality voting rule, in which voters can very frequently gain by voting for a 'compromise' candidate B other than their sincere preference A, thus preventing the election of a third candidate C whom they like less than both.[3] When used successfully in one particular election, this strategy may either increase or decrease utilitarian efficiency, depending on whether the compromise candidate happens to provide a higher sum of utilities than the original winner. However, if this strategy is employed habitually, it becomes extremely difficult for a third party to mount an effective challenge to an established pair of major parties. This can prevent a superior third-party candidate from being elected, and it can reduce the incentive of elected politicians to perform well, by reducing the competition they expect to face in future elections.

Next, consider the Borda rule, one of the oldest alternatives to plurality. In addition to the 'compromising' strategy that is endemic to the plurality rule, it is potentially vulnerable to a 'burying' strategy.[4] That is, in some cases voters are able to cause their favorite candidate A to win by insincerely ranking A's closest rival B below a third candidate C.[5] Successful use of this strategy is likely to reduce utilitarian efficiency. Further, the long-run consequences of burying strategies are not well understood, because commonly used single-winner election systems such as plurality, runoff, and Hare don't create incentives for burying.[6]

High manipulability in a voting rule is a source of concern not only because the winner of a particular election may be changed from one candidate to another, but also because voters' true preferences are less likely to be accurately revealed. This reduces the extent to which the election system fulfills an important function (facilitating the communication of popular will), and reduces the legitimacy of election outcomes.

2.2. Utilitarian efficiency

In this paper we define utilitarian efficiency operationally as the likelihood that sincere voting will result in the election of the candidate who maximizes the sum of the voters' utilities. To the extent that we view elections as a way to select the 'best' candidate, utilitarian efficiency is a logical criterion to consider, because maximizing the sum of utilities is a plausible operational definition of being 'best'.

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[3] For example, take an election in which 20 voters have preferences $A \succ B \succ C$, 35 voters have preferences $B \succ A \succ C$, and 45 voters have preferences $C \succ B \succ A$. In plurality, the first group of voters can change the winner from C to B by voting for B.

[4] Green-Armytage (2014) defines compromising and burying strategies, drawing from a framework developed in unpublished work by Blake Cretney.

[5] For example, take an election in which 45 voters have preferences $A \succ B \succ C$, 40 voters have preferences $B \succ A \succ C$, and 15 voters have preferences $C \succ B \succ A$. In Borda, the first group of voters can change the winner from B to A by voting $A \succ C \succ B$. The same can be said of minimax, and of several other rules that are equivalent to minimax in the three-candidate case.

[6] See Green-Armytage (2014).

However, voter utilities are not directly observable in practice, in part because of the possibility of strategic behavior. Therefore it is difficult (indeed, perhaps impossible) for a researcher to determine whether any particular candidate does in fact maximize the sum of utilities. Fortunately, we are not attempting here to evaluate voting *outcomes* but rather voting *rules*; that is, we are not focused on finding the true utilities of any electorate, but rather on determining whether some voting rules are more likely than others to achieve utilitarian efficiency. For this reason, we can legitimately bypass many of the more difficult epistemological issues inherent in utilitarian analysis by simply constructing simulated elections in which we know the utilities because we have generated them ourselves.

Thus, we reason that voting rules that are more likely to maximize the sum of utilities in simulated elections are also more likely to do so in actual elections, even though utilities can't be directly known in the latter as they can in the former. But to this, two important caveats must be added. First, the likelihood of each rule's success in simulations depends on the process used to generate voter preferences; for this reason we explore several such processes and look for results that are widely persistent. Second, the efficiency of voting rules may be impacted in practice by strategic behavior; this underlines the importance of considering efficiency in tandem with resistance to strategy, which is our next topic.

2.3. Complementarity of the two statistics

Evaluating voting rules simultaneously in terms of strategic resistance and utilitarian efficiency is valuable for at least two reasons. First, in order to measure a voting rule's utilitarian efficiency it is necessary to make some assumption about how voters will behave. The assumption that voters express their preferences sincerely is the simplest and most transparent one, but it is less plausible when applied to voting rules that frequently provide opportunities for voters to gain from *in*sincerity. Therefore, we argue that the utilitarian efficiency of a voting rule is more difficult to predict when that voting rule is highly manipulable.

Second, the juxtaposition of utilitarian efficiency and resistance to strategy allows us to illustrate some interesting tradeoffs. For example, with sincere voting the range voting rule[7] achieves perfect utilitarian efficiency under certain assumptions, but it is highly vulnerable to strategic manipulation that can be expected to undermine its efficiency. On the other hand, the random dictator rule[8] is entirely immune to strategy (and also encourages voters to report their first choices sincerely), but it performs very poorly in terms of utilitarian efficiency. Between these two voting rules, there will be a frontier of voting rules along which greater utilitarian efficiency can only be gained at the expense of less resistance to strategy, and vice versa; the shape of this frontier and the rules that lie along it are likely to be of interest to those who would like to identify the most suitable rule for any specified electorate.

[7] Voters rate the candidates on a closed scale, and the ratings are summed to determine the winner.
[8] One ballot is selected at random, and the candidate listed first on this ballot is elected.

3. Literature

3.1. Strategic voting literature

The literature on strategic voting is large and swiftly growing. A common starting point is the Gibbard-Satterthwaite theorem (Gibbard, 1973; Satterthwaite, 1975), which shows that no non-dictatorial single-winner voting rule with a universal domain, for elections with more than two candidates, can be free of incentives for strategic voting in all cases. This suggests that it may be valuable to compare voting rules in terms of vulnerability to strategy, measured quantitatively. The two most common such measures are (1) estimates of how frequently each rule gives individuals incentives to vote insincerely,[9] and (2) estimates of how frequently each rule gives coalitions incentives to vote insincerely.[10] In this paper, we focus on incentives for coalitions to vote insincerely, because we would like results that apply to large elections, where individual voters are rarely decisive.

Studies typically proceed by estimating the share of elections in which any specified rule is resistant to strategy. Therefore, the numerical scores assigned to each rule depend on assumptions made in the analysis about the relative likelihoods of different combinations of voter preferences. The relative likelihoods employed in a study may be generated by mathematical assumptions, or they may have an empirical source. Commonly employed mathematical assumptions include the 'impartial culture' (IC) model,[11] the 'impartial anonymous culture' (IAC) model,[12] and spatial models.[13] The use of empirical data is somewhat less common in the literature, but it has been done by Tideman (2006, chapter 13), using data from private elections, and by Green-Armytage (2014), using American National Election Studies (ANES) survey data.

3.2. Utilitarian efficiency literature

The literature on utilitarian efficiency is smaller than the literature on strategic voting. Two measures of efficiency are predominant here: the share of trials in which the candidate that maximizes the sum of utilities is chosen, and the average share of available welfare that is produced by the chosen candidate. Either analysis requires a contestable process for generating cardinal values of individual utility. Weber (1978) generates voter utilities as independent draws from a uniform distribution. Bordley (1983) generates them as draws from uniform and normal distributions, but allows for correlations among voters' preferences. Merrill (1984) provides one set of results using Weber's method, and another set using a spatial model. Merrill and Tideman (1991) derive 96 synthetic elections from thermometer scores for

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[9] See, for example, Nitzan (1985), Smith (1999), and Aleskerov and Kurbanov (1999).

[10] See, for example, Chamberlin (1985), Lepelley and Mbih (1994), Kim and Roush (1996), Lepelley and Valognes (2003), Favardin and Lepelley (2006), Pritchard and Wilson (2007), and Green-Armytage (2014).

[11] Chamberlin (1985), Kim and Roush (1996), Lepelley and Valognes (2003), Pritchard and Wilson (2007) and Green-Armytage (2014) use the IC process to measure voting rules' resistance to coalitional manipulation.

[12] Lepelley and Mbih (1994), Favardin et al (2002), Lepelley and Valognes (2003), Favardin and Lepelley (2006), and Pritchard and Wilson (2007) use the IAC process to measure voting rules' resistance to coalitional manipulation.

[13] Chamberlin (1985) and Green-Armytage (2014) use spatial models to measure voting rules' resistance to coalitional manipulation.

presidential candidates, drawn from 1972-1984 data from the ANES. The number of voting rules considered in these studies ranges from three to seven.

Apesteguia et al. (2011) take a more theoretical approach, and find that some positional rule must always surpass all other ranked ballot rules in terms of the expected sum of voter utilities from the winning candidate. However, this result depends on the assumption that each voter's utility from each candidate is an independent and identically distributed random variable, which implies that each ranking of the candidates is equally probable for each voter. In other words, the authors' model has the same implications as the IC model. Although IC has been used many times in the literature (perhaps in part because it is very simple and thus lends itself to the derivation of analytical results) it is widely acknowledged to be unrealistic.[14] For example, it implies that plurality elections with large numbers of voters will generally result in all candidates getting vote shares that are very close to equal, which is dramatically at variance with what we observe empirically. Thus, we believe that the best way to carry forward the discussion of the relative efficiency of voting rules is to consider a wider range of data-generating processes.

4. Computation

Each of our data sources allows us to construct many synthetic three-candidate elections. For each data source and each voting rule under consideration, our two tasks are to determine, (1) the share of these synthetic elections in which there is no possibility of strategic voting, and (2) the share of these synthetic elections in which the candidate who maximizes the sum of voter utilities is elected. We refer to these statistics as R (for 'resistance' to strategy) and E (for 'efficiency') respectively.

We limit ourselves to three-candidate elections because it increases the tractability of our strategic voting analysis significantly. Furthermore, Green-Armytage (2014) provides evidence that the ranking of voting rules in order of resistance to strategy is largely preserved when the number of candidates increases from three; this suggests that studies with larger numbers of candidates are likely to agree with ours on the question of which voting rules have the highest and lowest resistance scores.

4.1. The choice of a pair of measures

Many other measures of resistance to strategy and utilitarian efficiency are possible; we select these particular measures not because we believe that they are the only ones worth pursuing, but because we believe that they provide the most logical starting points for combined analysis. We do not intend this paper to close the discussion on evaluating voting rules jointly in these dimensions, but rather to open it, in the hope that others will develop alternative methodologies and produce complementary results.

[14] As noted e.g. by Tsetlin et al. (2003).

As for other measures of utilitarian efficiency, we also collected results using the other main measure in the literature (the average ratio of the winning candidate's sum of utilities to the maximal value of the sum of utilities), but we find that the difference between the results with this measure and those with our measure (the frequency with which the candidate maximizing the sum of utilities is chosen) is not great enough to justify including both here. Having decided to present only one of these, we opted for the latter because of its symmetry with our R measure.

As for other measures of resistance to strategy, we recognize that several interesting alternatives are possible; we have explored some of these in previous work, and plan further exploration in a future paper. For example, one may assign weights to possible strategic outcomes according to the maximum utility loss that might result, according to the coalition size they require (as a percentage of all voters, or as a percentage of the voters who share the coalition's interest), according to the strategy's margin of error (including the potential risk that the strategy might backfire), or according to the ease with which strategies can be blocked by other voters. Alternatively, one may suppose that only voters with a strong-enough preference for an alternative candidate will choose to strategize, or rule out coalitions that are unstable in the sense of containing some members who may be tempted to cause the election of another alternative candidate whom the other coalition members do not like. And so on. Meanwhile, our current R measure addresses the question of manipulability in a simple, transparent way, which does not require any additional parameters beyond those assumed by the data-generating process. Furthermore, it is in our estimation the most commonly-used measure of resistance to strategy in the recent voting literature,[15] which makes it a logical touchstone to use as we begin to develop joint analyses of resistance to strategy and utilitarian efficiency.

4.2. Calculating resistance to strategy

Now we describe the measures we do perform in more detail, beginning with the measurement of strategic voting, which is the more difficult of the two. For each election, we must determine whether there is any candidate, q, other than the sincere winner, w, such that the voters who prefer q to w can change their ballots in any way and thereby change the winner to q.[16] R is defined as the share of elections in which this is not the case.

It is impractical to make this determination by brute computational force. Therefore we instead use algorithms that are tailored directly to individual voting rules or classes of voting rules (such as positional rules, etc.) under consideration. This is made substantially easier by our three-candidate assumption, but it still requires creative programming.[17]

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[15] For example, this measure is employed by Chamberlin (1985), Lepelley and Mbih (1994), Kim and Roush (1996), Lepelley and Valognes (2003), Favardin and Lepelley (2006), Pritchard and Wilson (2007), and Green-Armytage (2014), as we note in section 3.1 above.

[16] For example, suppose that there is an election in which 49 voters have preferences $A \succ B \succ C$, 48 voters have preferences $B \succ A \succ C$, and 3 voters have preferences $C \succ B \succ A$. Sincere voting under plurality chooses A, but those who prefer B to A can succeed in electing B by voting for B. Sincere voting under the Borda count chooses B, but those who prefer A to B can succeed in electing A by voting $A \succ C \succ B$. Sincere voting under Hare chooses B, and neither the voters who prefer A to B nor the voters who prefer C to B can do anything to elect their preferred candidate. Thus, this election provides an instance in which plurality and Borda are manipulable while Hare is not.

[17] Descriptions of our algorithms, and the codes themselves, are available on request.

4.3. Calculating utilitarian efficiency

Compared with the above, the calculation of utilitarian efficiency scores is straightforward once the synthetic elections have been generated. In each election, we find the candidate who maximizes the sum of voter utilities (as determined by thermometer scores or draws from assumed distributions). E is defined as the share of elections in which this candidate is elected.

4.4. Equal rankings and tie-breaking

In our calculations, we assume that each ballot must mention all available candidates. For all ranked ballot rules, we assume that equalities in reported rankings (which could arise from strategic voting, and which are present in sincere votes when the Politbarometer and ANES sources are used) are treated as the average of all strict rankings that can be formed by resolving expressed indifferences; for example, an $\mathrm{A} \succ \mathrm{B} \sim \mathrm{C}$ vote is treated as half an $\mathrm{A} \succ \mathrm{B} \succ \mathrm{C}$ vote and half an $\mathrm{A} \succ \mathrm{C} \succ \mathrm{B}$ vote.

When candidates receive the same score under a voting rule, we break the tie lexicographically.

5. Voting rules

We apply our analysis to 54 voting rules for elections with three candidates: eleven positional rules, eleven elimination rules, eleven Condorcet-positional rules, eleven Condorcet-elimination rules, three cardinal rules, four Condorcet-cardinal rules, two other rules based on pairwise comparisons, and one dictatorial rule. It is impossible to include every conceivable voting rule, but our selection is intended to be as comprehensive as reasonable space constraints will admit, and to include the rules that are most promising and most prominent in the literature.

5.1. Positional rules

A positional voting rule (or ‘scoring rule’) is a voting rule in which each position on each voter’s ballot (first choice, second choice, etc.) earns the indicated candidate a prescribed number of points, and the winner is the candidate who obtains the most points. Authors such as Saari (1994) have noted that in the case of three-candidate elections, all monotonic positional rules can be located on a spectrum from plurality to anti-plurality, with Borda in the middle. That is, we can assume without loss of generality that a first choice vote is worth one point, a third choice vote is worth zero points, and a second choice vote is worth $p \in [0, 1]$ points. If $p = 0$, we have the plurality rule: the winner is the candidate with the most first choice votes. If $p = 1$, we have the anti-plurality rule: the winner is the candidate with the fewest last choice votes. If $p = 1/2$, we have the Borda rule: the difference in points between a first and second choice vote is equal to the difference in points between a second and third choice vote.[18] We evaluate the eleven positional rules defined by $p = 0, 0.1, 0.2, \ldots, 1$.

[18] See Borda (1784).

Table 1: The 54 voting rules that we consider

positional	elimination	Condorcet-positional	Condorcet-elimination	straight cardinal	Condorcet-cardinal	other comparison-based	dictatorial
$p = 0$ (plurality)	$p = 0$ (Hare[a])	$p = 0$ (Condorcet-plurality)	$p = 0$ (Condorcet-Hare[b])	range	Condorcet-range	minimax[c]	random dictator
$p = .1, \ldots, .4$	$p = .1, \ldots, .4$	$p = .1, \ldots, .4$	$p = .1, \ldots, .4$	normalized range	Condorcet-normalized range	Nanson	
$p = .5$ (Borda)	$p = .5$ (Baldwin)	$p = .5$ (Black)	$p = .5$ (equivalent to Baldwin)	approval	cardinal pairwise		
$p = .6, \ldots, .9$	$p = .6, \ldots, .9$	$p = .6, \ldots, .9$	$p = .6, \ldots, .9$		normalized cardinal pairwise		
$p = 1$ (anti-plurality)	$p = 1$ (Coombs)	$p = 1$ (Condorcet-anti-plurality)	$p = 1$ (Condorcet-Coombs)				

[a] Hare is also known as the alternative vote, instant runoff voting and ranked choice voting.
[b] In the three-candidate case, Condorcet-Hare is equivalent to alternative Smith. The alternative Schwartz rule differs only in the treatment of pairwise ties.
[c] Minimax is also known as Simpson (or Simpson-Kramer). For three-candidate elections, it is equivalent to the Kemeny (or Kemeny-Condorcet-Young), Young, ranked pairs, beatpath (or Schulze), and simplified Dodgson voting rules.

5.2. Elimination rules

Favardin and Lepelley (2006) consider an analogous continuum of 'elimination' (or, 'iterative scoring') rules, from Hare to Coombs, with Baldwin in the middle. These rules conduct rounds of counting in which the candidate with the lowest score according to a positional rule is eliminated (and the number of places on each ballot is reduced by one), until a single winning candidate remains. In our three-candidate case, this is equivalent to eliminating the loser of a positional rule and then selecting between the remaining two candidates by majority rule. Thus the elimination rule with $p = 0$ is the Hare rule, which eliminates the plurality loser in each round until one candidate remains.[19] Likewise, the elimination rule with $p = 1/2$ is the Baldwin rule, which eliminates the Borda loser in each round,[20] and the elimination rule with $p = 1$ is the Coombs rule, which eliminates the anti-plurality loser in each round.[21] As with the positional rules, we evaluate the eleven elimination rules defined by $p = 0, 0.1, 0.2, \ldots, 1$.

5.3. Condorcet-positional and Condorcet-elimination rules

In a similar manner, we define two more continua: one continuum of 'Condorcet-positional' rules (with Black in the middle), and another of 'Condorcet-elimination rules,' with Condorcet-Hare at one end, and an equivalent to Baldwin in the middle.

[19] This rule is also known as the alternative vote, instant runoff voting, and ranked choice voting. Thomas Hare proposed transferring votes from plurality losers as a refinement of the single transferable vote procedure of proportional representation. See Hoag and Hallett (1926, 162-95).
[20] See Baldwin (1926).
[21] See Coombs (1964). Some versions of Coombs include a provision to automatically elect a candidate who holds a majority of first choice votes, but we do not use this provision here.

A ‘Condorcet winner’ is a candidate who, according to the ballot data, would win a two-person race against any other candidate. The Condorcet-positional rules elect the Condorcet winner if one exists, and otherwise elect the winner of the base positional rule. For $p = 1/2$, this becomes the Black rule, which elects the Borda winner when there is no Condorcet winner.[22] The Condorcet-elimination rules elect the Condorcet winner if one exists, and otherwise revert to an elimination rule. For example, $p = 0$ yields the Condorcet-Hare rule, which elects the Hare winner when there is no Condorcet winner.[23] $p = 1/2$ yields ‘Condorcet-Baldwin’, but since Baldwin (alone among the elimination or positional rules) is already Condorcet-consistent,[24] this is equivalent to Baldwin itself. Again, for each of these two additional continua, we evaluate the eleven rules defined by $p = 0, 0.1, 0.2, \ldots, 1$.

5.4. Cardinal rules and Condorcet-cardinal rules

A cardinal rule is a rule that is based on ratings of candidates according to a specified scale. A straight cardinal rule is a cardinal rule that does not have any non-cardinal component. The first straight cardinal rule, range voting, elects the candidate with the greatest sum of ratings. Normalized range voting is the same, except that it scales the ratings reported by voters so that, for each voter, the highest is 1, the lowest is 0, and the middle one has the same relative position as in the original data. (For example, ratings of 5, 3, and 2 become 1, 1/3, and 0.) Approval voting is the straight cardinal rule that restricts voters to giving each candidate a rating of either one or zero, and again the candidate with the greatest sum of ratings wins.[25] The method we use to derive ‘sincere’ approval votes from our data-generating processes is to assume that each voter gives ratings of one to each candidate who offers average or above-average utility and gives ratings of zero to the others.[26]

Next, we define the Condorcet-cardinal rules. These are rules that combine Condorcet and cardinal components. The simplest Condorcet-cardinal rule is Condorcet-range, which elects the Condorcet winner if there is one and otherwise elects the range winner. A variant of this rule is Condorcet-normalized range, which elects the normalized range winner if there is no Condorcet winner. We also evaluate two “cardinal pairwise” rules. These rules work like minimax, except that if there is no Condorcet winner, the strength of the defeat of one candidate, B, by another, A, is defined as the sum of the differences in ratings of A and B by the voters who rate A above B.[27] As with range and Condorcet-range, we evaluate both non-normalized and normalized versions of this rule.

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[22] Black (1958) creates and advocates this rule.

[23] Green-Armytage (2011) examines four Condorcet-Hare hybrid rules (proposed separately by different voting theorists) that can occasionally yield different results when there are four or more candidates; these include ‘alternative Smith’ as defined in Tideman (2006). However, since they always yield the same result when there are three or fewer candidates, we can treat them as one rule for present purposes.

[24] As Nanson (1882) explains, a Condorcet winner always has an above-average Borda score. Therefore the Baldwin rule can never eliminate such a candidate, which means that a Condorcet winner must also be a Baldwin winner.

[25] See Brams and Fishburn (1978).

[26] Here, ‘average utility’ means the average of the individual voter’s utilities from the three candidates. There is no general agreement in the literature on what it means to sincerely ‘disapprove of’ a candidate, but the idea that it means liking the candidate less than average seems as straightforward as any.

[27] See Green-Armytage (2004).

5.5. Other comparison-based rules

A comparison-based rule is a rule under which it is possible to determine the winner solely from the results of the paired comparisons of the candidates. The Borda, Black and Baldwin rules could be placed in this category if they had not already arisen in other categories. Thus the 'other comparison-based' category consists of comparison-based rules that are not elsewhere classified. The first such rule we consider is minimax; this rule assigns each candidate a score according to the greatest margin by which the candidate loses to another candidate in a pairwise comparison, and then chooses the candidate with the lowest score.[28] There are at least five other voting rules that, for elections with three candidates, are equivalent to minimax; these rules are Kemeny (also known as Kemeny-Condorcet-Young), Young, ranked pairs, beatpath (also known as Schulze), and simplified Dodgson.[29]

The Nanson rule is also comparison-based. It is similar to the Baldwin rule, except that instead of eliminating the single candidate who has the lowest Borda score in each round, it eliminates all candidates at once who have average or lower-than-average Borda scores.[30]

5.6. The random dictator rule

Finally, we evaluate the random dictator rule. In this rule a ballot is chosen at random and the candidate at the top of that ballot is elected. Our implementation for the purpose of measuring utilitarian efficiency assigns each candidate a probability of being elected equal to its share of first-choice votes.

6. The effects of a Condorcet provision

We evaluate a number of voting rules that are not Condorcet consistent, and for nearly all of these we also evaluate a corresponding composite voting rule that elects the Condorcet winner when one exists and elects the winner of the base rule otherwise. This allows us to give general consideration to the effects of such a provision – i.e. a 'Condorcet provision' – within our evaluative framework.

6.1. Effects on utilitarian efficiency

When an election has a Condorcet winner, all Condorcet-consistent rules will choose the same candidate. Therefore, when the likelihood of a sincere Condorcet winner is very high, as is the case in most of the data-generating processes we employ, all Condorcet-consistent rules will have very similar E scores, which will be approximately equal to the probability that the Condorcet winner is also the candidate who maximizes the sum of utilities.[31] Thus, the addition of a Condorcet provision may either increase or decrease the E score of a base rule, depending on whether its prior score was higher or lower than this probability; overall, the effect is a 'flattening-out' to a reasonably high efficiency rate.

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[28] Black (1958, p. 175) develops the minimax method as a possible interpretation of Condorcet's proposal. It has also been variously referred to as 'Simpson', 'Simpson-Kramer', 'successive reversal', and 'maximin'.

[29] For discussion of these rules see Tideman (2006) pp. 182-90, 199-201, 217-223 and 228-232.

[30] See Nanson (1882). Like Baldwin, this rule is Condorcet-consistent, so 'Condorcet-Nanson' would be redundant.

[31] This probability depends on the data-generating processed used, the number of candidates and voters, etc. In most of the models that we employ here, it is above 90%; the two exceptions to this are our implementation of the IAC and IC models.

6.2. Effects on resistance to strategy

For resistance to strategy, on the other hand, we show here that it is *not* possible for the addition of a Condorcet provision to cause a decrease in the R score of any base rule belonging to a broad category, i.e. the category of rules possessing a property that we call 'conditional majority determination'. That is, for any rule that possesses this property, and any election for which that rule is immune to strategy, the rule with a Condorcet provision added must also be immune to strategy. This central result is given below as proposition 2. Proposition 1 lays groundwork for this by clarifying which positional rules possess conditional majority determination, while proposition 3 demonstrates that each elimination rule with $p \leq 1/2$ has an R score that is equal to that of its corresponding Condorcet-elimination rule – a special case. While proposition 2 is general with respect to the number of candidates, propositions 1 and 3 focus exclusively on three-candidate elections.

To be clear, the results in this section pertain only to the *logical possibility* of manipulation in a voting rule. The frequency with which manipulation actually occurs is a different and substantially more difficult question, as is the expected loss in welfare associated with strategic voting. We reserve these more difficult questions for future inquiry, while suggesting in the meantime that it is reasonable, *ceteris paribus* (note the emphasis), to expect a greater frequency of the possibility of manipulation to be associated with more frequent manipulation and a higher social cost of manipulation.

6.2.1. The CMD property

The conditional majority determination (CMD) property is defined as follows: If, when rule X is used, a group comprising a majority of voters is always able to cast their votes in such a way as to elect any candidate they wish, provided that the votes of the remaining minority are known to them and held constant, then rule X possesses CMD.

It is easy to see that plurality possesses this property, as a majority can elect any candidate by all ranking that candidate in first position. Borda possesses CMD as well; to see this, suppose that majority voters counter each of the minority voters' ballots with a ballot expressing exactly the opposite preferences (for example, $\mathrm{A} \succ \mathrm{B} \succ \mathrm{C}$ is countered by $\mathrm{C} \succ \mathrm{B} \succ \mathrm{A}$), then cast their remaining votes in any order that ranks their chosen candidate first. Furthermore, the other (non-dictatorial) rules we consider apart from the positional spectrum all possess CMD: For any Condorcet-consistent rule, it is sufficient for the majority to simply rank their chosen candidate in first position. This works with many elimination rules as well; for the rest (such as Coombs) it is sufficient for the majority to agree on any strict ordering that ranks the chosen candidate first. In range voting and approval voting, it is sufficient for the majority to give the maximum rating to their chosen candidate while giving the minimum rating to all other candidates.

On the other hand, anti-plurality does not possess CMD. For example, suppose that the minority consists of 40 individuals who vote $\mathrm{B} \succ \mathrm{C} \succ \mathrm{A}$, while the majority consists of 60 individuals who wish to elect A; they are unable to accomplish this because some other candidate will inevitably have fewer than 40 last-place votes. Underlining the relevance of the CMD property to our analysis, it is not necessarily true that Condorcet-anti-plurality will be immune to strategy in all examples where anti-plurality is immune to strategy. To see this, consider a three-candidate example in which 48 voters have preferences $\mathrm{A} \succ \mathrm{B} \succ \mathrm{C}$, 49 voters have preferences $\mathrm{B} \succ \mathrm{C} \succ \mathrm{A}$, and 3 voters have preferences $\mathrm{C} \succ \mathrm{A} \succ \mathrm{B}$. Under the anti-plurality rule and sincere voting, B is the winner, and no coalition can change its votes so as to achieve a mutually preferred outcome. However, under the Condorcet-anti-plurality rule, B is still the sincere winner, but if all of the 51 voters who prefer A to B rank A in first position, A wins.

Restricting our attention to the three-candidate case, we can be more precise about which positional rules possess CMD; this is the subject of proposition 1 below.

Proposition 1: *Positional rules possess CMD if and only if $p \leq 1/2$.*

In summary, the only rules in our analysis that do not possess the CMD property are the random dictatorship rule and the positional rules that are strictly on the anti-plurality side of Borda. Thus proposition 2 below is widely applicable.

6.2.2. Adding a Condorcet provision to a CMD rule

Proposition 2: *If, for a profile Π of sincere voter preferences, sincere voting is a core equilibrium*[32] *in the voting game for a voting rule X that possesses the CMD property, then sincere voting must also be a core equilibrium for profile Π when the voting rule is Condorcet-X, which chooses the Condorcet winner when one exists and chooses the X winner otherwise.*

In other words, for almost all well-known voting rules, if there is an election where strategic manipulation is impossible, manipulation will still be impossible if the rule in use is modified by the addition of a Condorcet provision. Equivalently, if manipulation is possible with the provision added, it must also be possible without the provision.

6.2.3. The special case of elimination rules with $p \leq 1/2$

Since the elimination rules possess CMD, we know that their R scores cannot be higher than those of the corresponding Condorcet-elimination rules. However, with three candidates and $p \leq 1/2$, a special case arises in which the addition of a Condorcet provision doesn't affect the scores at all.

Proposition 3: *For $p \leq 1/2$ and any sincere preference profile for three candidates such that the elimination-p rule is manipulable, the corresponding Condorcet-elimination-p rule is also manipulable.*

[32] A core equilibrium with respect to voting is a situation in which no group of voters can achieve a mutually preferable result by changing their votes in cooperation with one another.

6.2.4. Summary of results on resistance to strategy

In this section, we have shown that a voting rule X that possesses CMD must have an R score that is less than or equal to its Condorcet-X counterpart, *regardless* of the data-generating process that is used. The scores are strictly equal if X is an elimination rule with $p \leq 1/2$ (or if X is Condorcet-consistent to begin with), while the Condorcet-X score is strictly greater (excepting extremely restrictive data-generating processes) if X is any other rule possessing CMD that we have examined. As for a voting rule X that does not possess CMD, it is theoretically possible for it to have a higher R score than its Condorcet-X counterpart; whether this will be observed in a simulation depends on the likelihood or rarity of the voter preference combinations that are needed for this. Thus, we will revisit this issue in section 9.2, as we discuss our simulation results.

7. Data

We generate election data using two survey sources and four mathematical models. The survey sources are Politbarometer and ANES, and the mathematical models are IAC, IC and two spatial models. This combination of data-generating processes is unusually broad, permitting insight into the question of which results can generalized beyond a single data-generating process or group of similar processes.

7.1. Survey-based data generation

The Politbarometer is a survey of the German electorate that has been conducted on an approximately monthly basis since 1977. On a scale from -5 to 5, respondents rate both parties and politicians (both elected and aspiring), but we use only the ratings of politicians.[33] The data we use include 610 surveys, from 1977 to 2008 (some months are skipped, while other months have more than one survey), with a median of 1012 respondents. The number of politicians rated ranges from 4 to 21 (so the number of synthetic elections per survey ranges from 4 to 1330), with a median of 11. Since 1991 there have been separate surveys of East and West Germany; we maintain this separation. We create synthetic elections using all three-candidate subsets of the rated politicians in each survey, which yields a total of 126,637 synthetic elections.

Data from the American National Election Studies (ANES) are similar to the Politbarometer data in that respondents give 'thermometer' ratings to politicians (primarily presidential contenders), on a scale in this case from 0 to 100.[34] Since there is only one survey per presidential election, the number of

[33] The question (translated from German) is phrased as, "Please tell me, again with the thermometer of $+5$ to -5, what you think of some political leaders. $+5$ means that you think a lot of the politician; -5 means that you think nothing at all of him. If you do not know a politician, you naturally do not need to grade him. What do you think of…" The data are available via http://www.gesis.org/en/elections-home/politbarometer/

[34] The question is phrased as, "I'd like to get your feelings toward some of our political leaders and other people who are in the news these days. I'll read the name of a person and I'd like you to rate that person using something we call the feeling thermometer. Ratings between 50 degrees and 100 degrees mean that you feel favorable and warm toward the person. Ratings between 0 degrees and 50 degrees mean that you don't feel favorable toward the person and that you don't care too much for that

observations is not as large as with the Politbarometer data. We use data from presidential election years from 1972 to 2008, giving us 10 separate observation-years. Over all years, the median number of respondents was 2017 and the median number of politicians rated was 8. From these data we construct 847 synthetic three-candidate elections.

For both data sets, each synthetic election is constructed using the subset of voters who rated all three of the candidates used in the particular synthetic election.[35]

We regard this methodology of constructing synthetic elections from survey data as offering a valuable complement to the more mathematically abstract data generating processes that are more common in the literature. It is not strictly empirical in the sense of allowing us to measure how well the voting rules performed in historical circumstances, but it can reasonably be called ‘quasi-empirical’, in that the simulated elections have an empirical origin. They are based on reported preferences of real people with respect to real politicians in circumstances where there were no obvious potential gains from reporting false preferences. We do not know the extent to which these survey responses represent inter-personally comparable utilities, but any lack of realism in this regard should be considered in the context of the other available data-generating processes, which each require the use of unverifiable assumptions, and which have an even fainter connection to observable preferences than treating thermometer scores as utilities.

Of course, we do not argue that any of our data give us independent draws from the one ‘true’ distribution of preference profiles. Indeed, there almost certainly is no such unique distribution, as these probabilities can be expected to vary with such things as the time, place, and purpose of an election. However, the deep similarities that can be observed in results from different data sets such as Politbarometer and ANES suggest that voter preferences exhibit persistent patterns. In fact, the measures of efficiency and resistance to strategy developed from these two surveys are very highly correlated with each other (94.6% for efficiency and 99.0% for resistance to strategy), as well as with results from other sources, especially the multi-dimensional spatial model and the IAC model.

7.2. IAC and IC models

The impartial anonymous culture (IAC) model is a method of producing ranked ballot data that assumes that every possible division of the voters among the possible preference orderings is equally likely. To implement this model computationally, we follow Berg (1985) and Lepelley and Valognes (2003), who show that IAC is equivalent to an urn experiment that begins with one ball of each color (one

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person. You would rate the person at the 50 degree mark if you don’t feel particularly warm or cold toward the person. If we come to a person whose name you don’t recognize, you don’t need to rate that person. Just tell me and we’ll move on to the next one.” The data are available via http://www.electionstudies.org/

[35] As a robustness check, we also ran simulations in which 99 respondents were randomly chosen (with replacement) from each possible three-candidate subset of each survey, thus creating synthetic elections of equal size to the other four data-generating processes described below. For the Politbaometer data, the resistance to strategy results from this alternative approach had a correlation of .9993 with our main approach, and the utilitarian efficiency results had a correlation of .9972. For the ANES data (averaged over ten repetitions to increase the number of elections from 847 to 8470), the analogous correlations were .9987 and .9848. Therefore, we consider this robustness check to be successful.

ballot with each ranking). Each time that a ball (ranking of the candidates) is drawn, it is replaced in its urn along with one additional ball of the same color (ballot with the same ranking of the candidates). With our focus on three-candidate elections, there are $3! = 6$ rankings. For the purpose of assessing utilitarian efficiency, or of executing any rule that depends on cardinal information (such as range voting or approval voting), it is necessary to associate cardinal preferences with the ordinal preferences provided by IAC. To determine each voter's utilities for the three candidates in an election, we take three draws from the standard normal distribution and assign them to the three candidates in such a way that the ranking determined by IAC is preserved.[36]

Our implementation of the impartial culture (IC) model is similar. This model assumes that each voter has an equal probability of ranking the three candidates in any of the six possible ways. We satisfy this assumption by generating each voter's vector of utilities for the three candidates as three independent draws from a standard normal distribution.

7.3. Spatial models

Finally, we use a simple spatial model[37] in which voters and candidates have the same multivariate normal distribution in an S-dimensional issue space, with zero covariances among the locations in different dimensions. The utility that a given voter assigns to a given candidate is defined as the additive inverse of the Euclidean distance between their locations in this space.

7.4. Parameters of the models

The IAC, IC, and spatial models, as described, each require the choice of a small number of parameters; the IAC and IC models require the numbers of candidates and voters, while the spatial model requires these along with the number of issue dimensions. In the interest of brevity, we present just one or two specifications for each process. We use 99 voters for IAC, IC, and spatial model simulations.[38] We present two versions of the spatial model: one with one dimension and one with eight dimensions.[39] For all three of these models, each of our data points represents the average from 30,000 elections.

[36] In our IAC, IC, and spatial models, utilities are converted to sincere ratings on the allowed $[0, 1]$ ballot range in a linear manner, such that the highest utility of any voter for any candidate is mapped to a rating of 1, and the lowest utility of any voter for any candidate is mapped to a rating of 0.

[37] Chamberlin and Cohen (1978) and Green-Armytage (2014) use similar models.

[38] We've also gathered R and E statistics with larger electorate sizes for selected voting rules, using each of our six data-generating processes, including the randomized version of the survey-based processes as described in footnote 34. Preliminary results suggest that increasing the electorate size from 99 has only a very modest impact on the results, but we lack the space here to develop an authoritative treatment of the subject.

[39] While increasing the dimensionality of the spatial model beyond this point continues to have an impact on manipulability scores, the marginal impact diminishes as the number of dimensions increases. Thus we choose $S = 1$ and $S = 8$ to represent the extreme case of a strictly one-dimensional space as well as an example of higher-dimensional spaces in general. Although it is true that any spatial example with more than $C - 1$ dimensions can be represented as a $(C - 1)$-dimensional example, when a generating process with random uncorrelated coordinates for candidates is used, increasing the dimensionality of the space beyond $C - 1$ has an impact on manipulability scores.

Table 2: Resistance to strategy and utilitarian efficiency results, from Politbarometer data

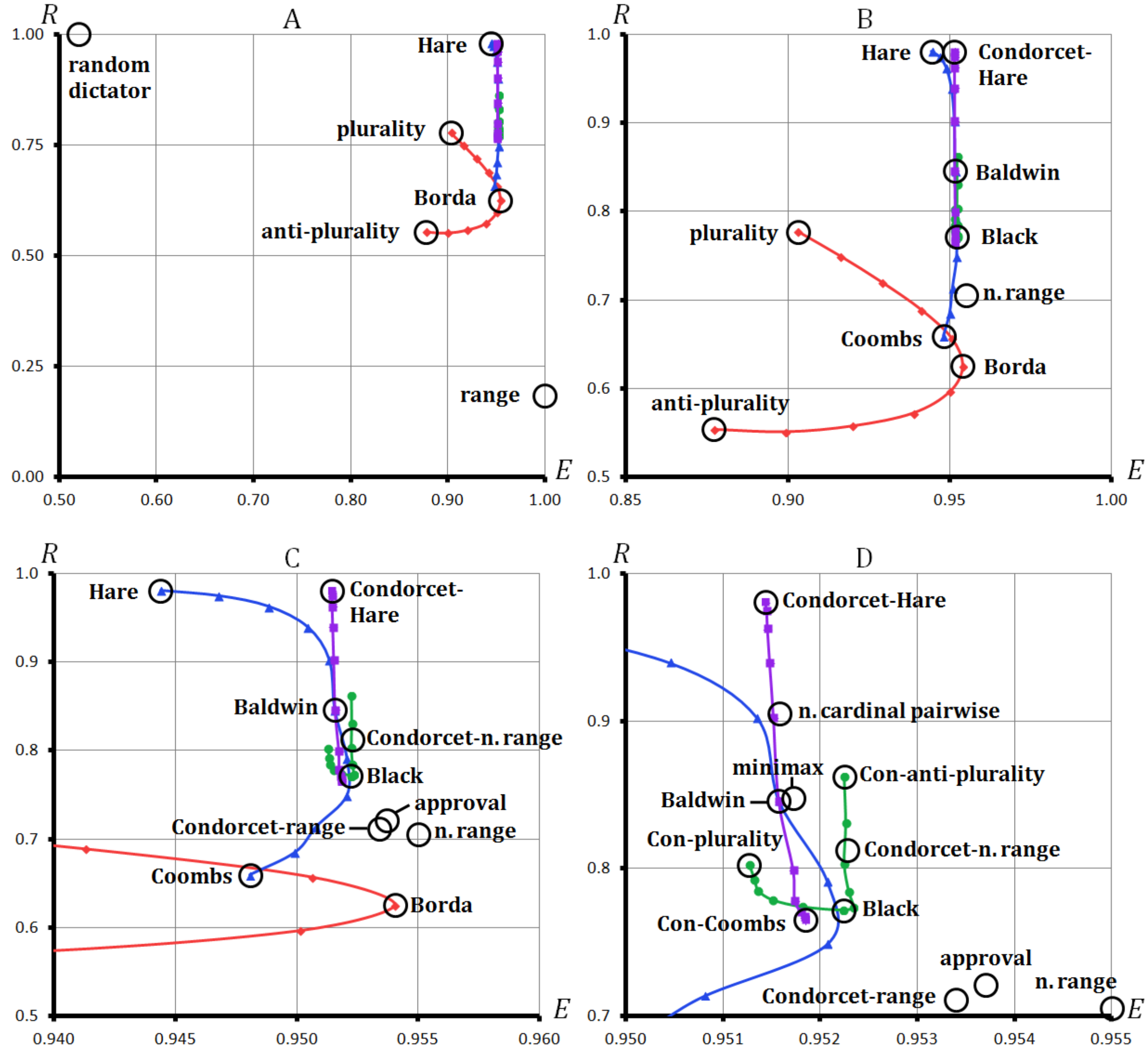
Voting rule	R	E	Voting rule	R	E
positional 0 (plurality)	.7749	.9027	Con-posit 0 (Con-plurality)	.8003	.9515
positional .1	.7474	.9159	Con-posit .1	.7904	.9516
positional .2	.7179	.9289	Con-posit .2	.7826	.9516
positional .3	.6864	.9414	Con-posit .3	.7762	.9518
positional .4	.6546	.9508	Con-posit .4	.7716	.9520
positional .5 (Borda)	.6226	.9542	Con-posit .5 (Black)	.7692	.9524
positional .6	.5937	.9498	Con-posit .6	.7713	.9525
positional .7	.5700	.9380	Con-posit .7	.7820	.9524
positional .8	.5547	.9197	Con-posit .8	.8013	.9524
positional .9	.5481	.8991	Con-posit .9	.8288	.9524
positional 1 (anti-plurality)	.5513	.8766	Con-posit 1 (Con-anti-plurality)	.8606	.9524
elimination 0 (Hare)	.9804	.9445	Con-elim 0 (Con-Hare)	.9804	.9516
elimination .1	.9743	.9469	Con-elim .1	.9743	.9516
elimination .2	.9622	.9490	Con-elim .2	.9622	.9516
elimination .3	.9388	.9506	Con-elim .3	.9388	.9516
elimination .4	.9012	.9515	Con-elim .4	.9012	.9517
elimination .5 (Baldwin)	.8441	.9517	Con-elim .5 (Baldwin)	.8441	.9517
elimination .6	.7893	.9518	Con-elim .6	.7968	.9519
elimination .7	.7466	.9518	Con-elim .7	.7760	.9519
elimination .8	.7120	.9512	Con-elim .8	.7683	.9520
elimination .9	.6828	.9503	Con-elim .9	.7646	.9520
elimination 1 (Coombs)	.6572	.9485	Con-elim 1 (Con-Coombs)	.7627	.9520
Range	.1814	1.0000	Condorcet-range	.7084	.9535
normalized range	.7029	.9550	Condorcet-normalized range	.8105	.9524
cardinal pairwise	.7205	.9519	normalized cardinal pairwise	.9042	.9517
approval	.7188	.9537	Minimax	.8461	.9519
random dictator	1.0000	.5181	Nanson	.8464	.9519

8. Results from the Politbarometer data

In this section we present results from the Politbarometer data source, and in the next section we present results from the other five data sources. We choose to begin with a data-generating process based on survey results because this methodology is more novel to the literature than the use of mathematical models, and we choose to begin with the Politbarometer surveys in particular because they allow us to construct far more observations than the ANES surveys.

Table 2 presents the Politbarometer-based results for resistance to strategy (R) and utilitarian efficiency (E). Figure 1 displays these results as a series of scatterplots in progressively greater detail, with E on the x-axis and R on the y-axis, and with lines connecting voting rules that are members of the same continuum. Panel A shows the widest picture of the data: random dictator has 100% resistance to strategy but just under 52% utilitarian efficiency, while range voting has 100% utilitarian efficiency but is vulnerable to manipulation in over 80% of cases.

Figure 1: Resistance to strategy and utilitarian efficiency results, from Politbarometer data



Our remaining rules are clustered near the upper right corner of panel A, so panel B zooms in with respect to both the E dimension and the R dimension to take a closer look. Here we find that the elimination, Condorcet-positional, and Condorcet-elimination rules all have similar utilitarian efficiency, but that they differ greatly in resistance to strategy: Hare and Condorcet-Hare are least likely to be manipulable, with R scores of about 98%, Baldwin and Black are intermediate, with R scores of about 84% and 77%, respectively, and Coombs is most likely to be manipulable, with an R score of about 66%.

The positional rules differ substantially from one another in both utilitarian efficiency and resistance to strategy. Of these, plurality is the least manipulable (although it is still far more manipulable than rules like Hare and Condorcet-Hare). As p (the value of a second choice vote) increases, we lose resistance to strategy, but we gain utilitarian efficiency. This continues until we reach Borda, which has the highest utilitarian efficiency among the positional rules we examine. However, increasing p from 0.5 to 0.6 decreases both R and E, and although R eventually increases again when p increases from 0.9 to 1.0, the positional rules with $p \in [0.6, 1.0]$ are all dominated by Borda.

In panel C, we zoom in again along the E dimension, and we continue to see that the Condorcet-consistent rules have very similar E scores, which happens because over 99% of the synthetic elections generated from the Politbarometer data have a Condorcet winner.[40] The rules near the middle of the elimination continuum have E scores that are similar to their Condorcet-elimination counterparts, which is logical because the two continua share a common midpoint, Baldwin. However, the rules close to the endpoints of the elimination continuum (Hare and Coombs) have lower E scores. Here we can see distinctly a mild advantage of Condorcet-Hare over Hare.[41]

Panel D zooms in even further along both dimensions, to focus on the region occupied by the Condorcet-consistent rules. We see here that the Condorcet-positional rules with $p \in [0.5, 1.0]$ have the highest E scores among Condorcet-consistent rules, aside from Condorcet-normalized range and Condorcet-range, the latter of which defines the upper limit in this category. However, although these rules have higher E values than Condorcet-Hare, the difference is very slight (about 0.2% at most), while the difference in resistance to strategy is relatively large (at least 11%). Similarly, Borda, approval, and normalized range have E values that are higher still by up to a few tenths of a percentage point, but at the expense of a greater loss in R. Therefore Condorcet-Hare is the most attractive rule in terms of the combination of these measures, unless one weights the measure of utilitarian efficiency (under the assumption of non-strategic voting) much more heavily than the measure of resistance to strategy.

9. Comparing results across data sources

Figures 2-4 each consist of six panels, each giving results according to one of our six data-generating processes: the Politbarometer data, the ANES data, the IAC model, the IC model, the eight-dimensional spatial model, and the one-dimensional spatial model. Figure 2 gives R (resistance to strategy) as a function of p (the value of a second choice vote) for each of the four voting-rule continua we study. Figure 3 gives E (utilitarian efficiency) as a function of p, for each of the same four continua. Figure 4 gives a series of ER scatter plots, like the ones in Figure 1. Table 3 shows the correlations between the R scores derived from different data-generating processes, and Table 4 does the same for the E scores.

9.1. Similarity and dissimilarity among data sources

Our first observation is that the Politbarometer data, ANES data, eight-dimensional spatial model, and IAC model produce remarkably similar R results, and that the first three of these produce remarkably similar E results. If we consider only these four models, the average correlation between pairs of R scores

[40] This is consistent with Feld and Grofman (1992), who analyze a data set of private elections conducted using STV, and find that fewer than 0.5% of all possible three-candidate subsets create majority-rule cycles.

[41] Green-Armytage (2011, 2014) shows another potentially important advantage: Hare is more likely than Condorcet-Hare to provide incentives for candidates to strategically exit races, which can have the effect of depressing the number of candidates, and reducing the competitiveness of elections. To see why this is the case, consider that if there is a Condorcet winner, and the voting rule used is Condorcet-consistent, the removal of any non-winning candidate from the ballot cannot change the outcome.

Figure 2: Resistance to strategy results, from six data sources

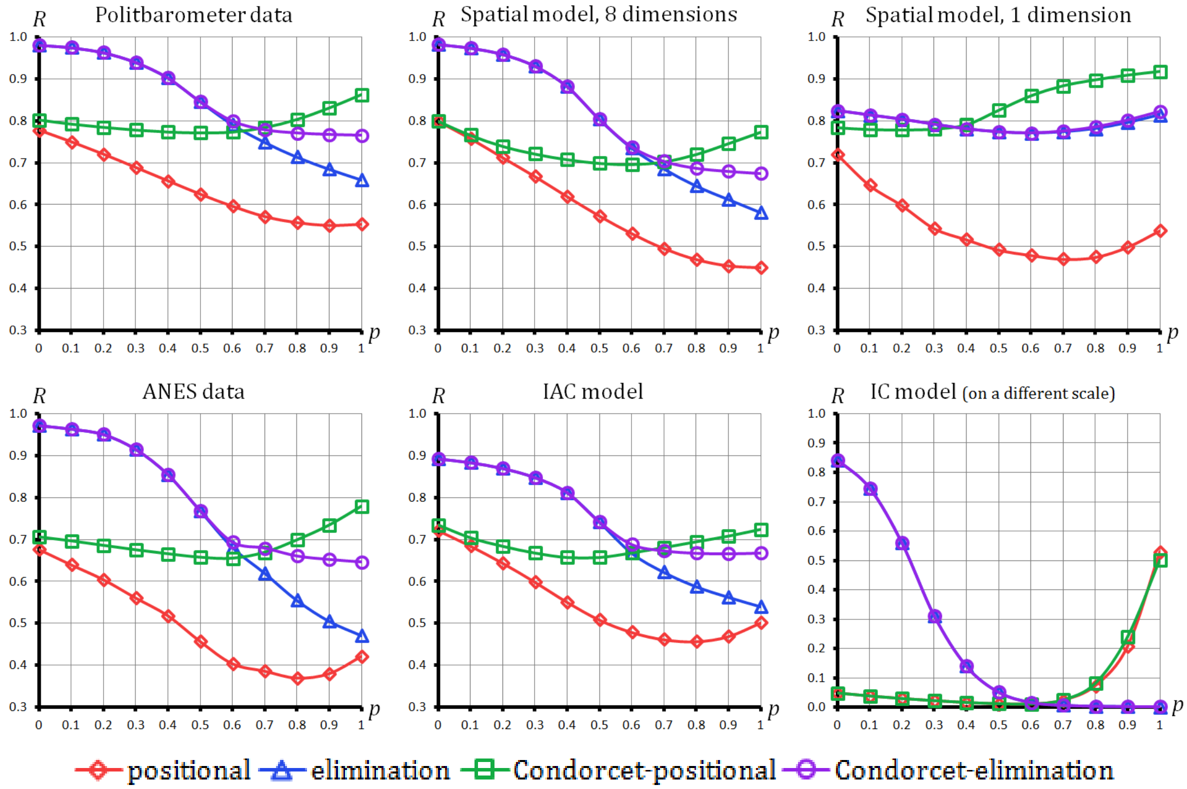
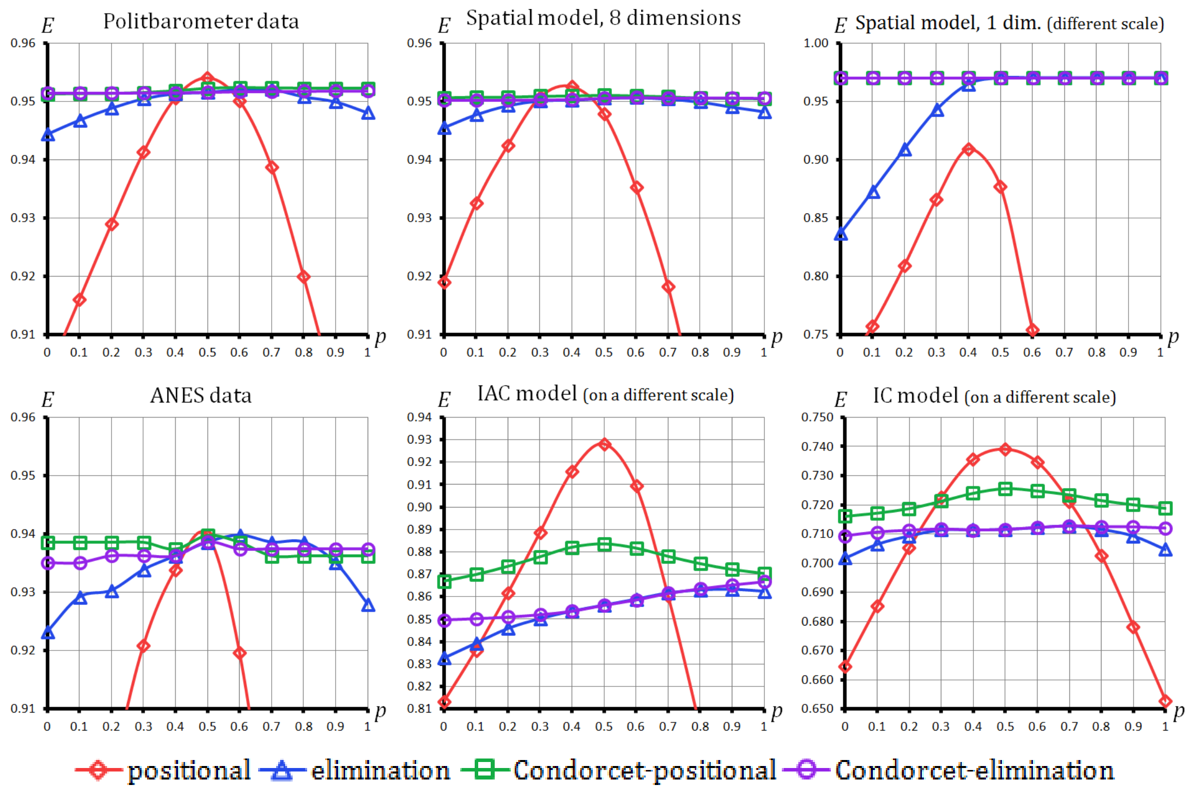
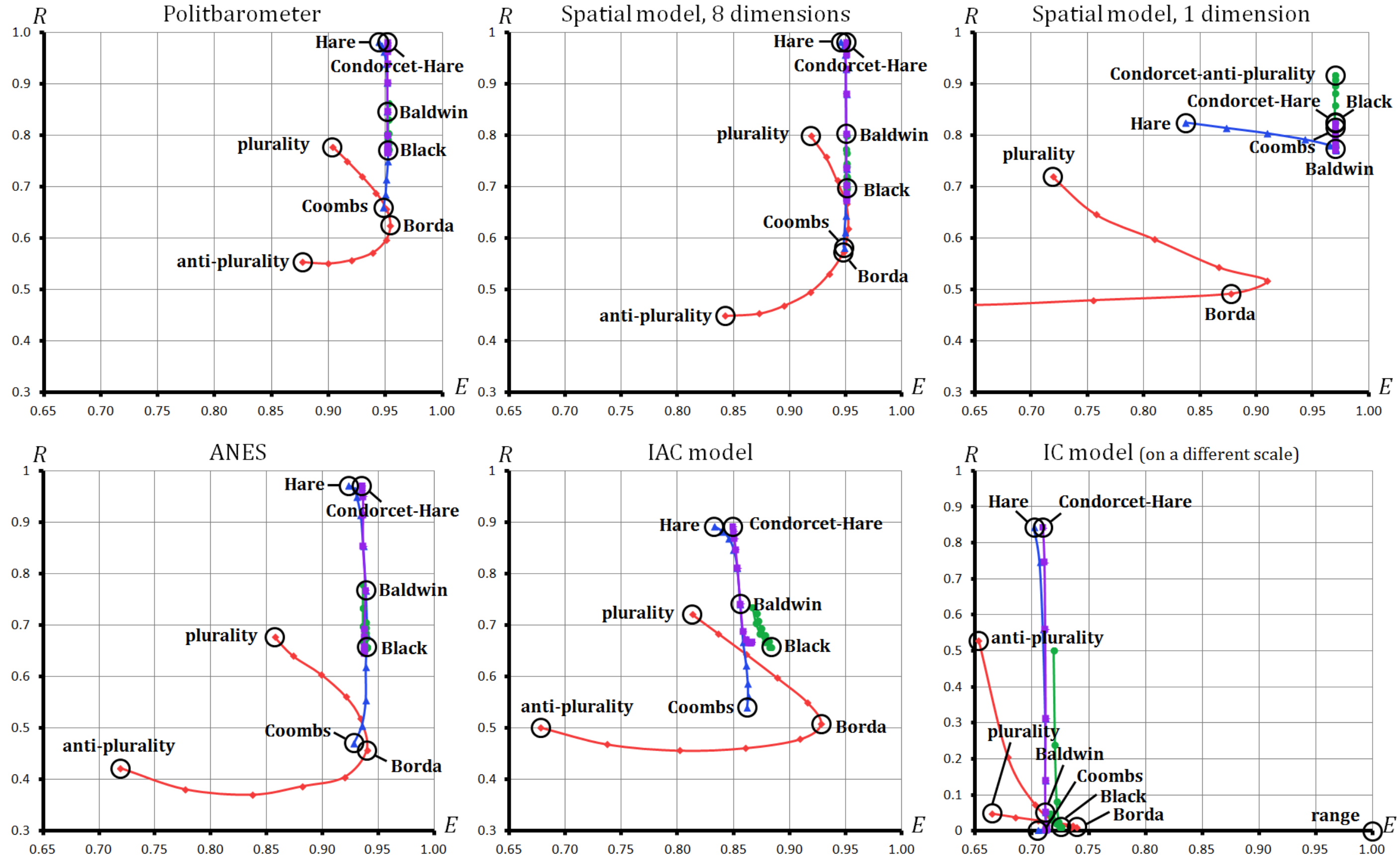


Figure 3: Utilitarian efficiency results, from six data sources.



Each panel shows resistance to strategy (R, in figure 2) or utilitarian efficiency (E, in figure 3) scores of voting rules along each of the four continua we study, according to one of six data-generating processes. The parameter p on the horizontal axes represents the value of a second choice vote.

Figure 4: Resistance to strategy and utilitarian efficiency results, from six data sources.



Each point in each panel represents a voting rule, plotted according to utilitarian efficiency (E) and resistance to strategy (R), using one of six data-generating processes. Voting rules on the same continua are connected by lines, and selected voting rules are labeled. Results from the Politbarometer, ANES, and eight-dimensional spatial data-generating processes have the most similar shapes. Results from the IAC process are very similar as well, particularly in the R dimension. The one-dimensional spatial model and IC model produce more idiosyncratic results.

Table 3: Resistance to strategy: correlations between data-generating processes

	Polit.	ANES	Spatial-8	IAC	Spatial-1	IC
Polit.		.990	.810	.979	.810	.566
ANES			.977	.972	.761	.623
Spatial-8				.988	.778	.587
IAC					.826	.605
Spatial-1						.331

Table 4: Utilitarian efficiency: correlations between data-generating processes

	Polit.	ANES	Spatial-8	IAC	Spatial-1	IC
Polit.		.946	.987	.862	.589	.841
ANES			.975	.921	.785	.807
Spatial-8				.890	.670	.833
IAC					.640	.876
Spatial-1						.471

The dotted lines emphasize that there is greater consonance among the Politbarometer, ANES, eight-dimensional spatial, and IAC processes than with the one-dimensional spatial and IC models. All of the 54 voting rules that we study are included in these calculations.

is 96.7%, and the minimum correlation is 81.0%. Likewise, the average correlation between pairs of E scores is 93.0%, and the minimum correlation is 86.2%. On the other hand, although they have some features in common with the other results, the results from the one-dimensional model and the IC model are outliers in many respects. The results from the one-dimensional spatial model have average correlations of just 79.4% and 67.1% with the results from the first four data sources in the R and E dimensions, respectively. Likewise, the IC model results have average correlations of just 59.5% and 83.9% with the results from the first four data sources.

Thus, in the process of gathering information about voting rules' resistance to strategy and utilitarian efficiency, we have gathered some indirect measures of the degree to which different data generating processes are similar to one another. Again, we do not argue for the existence of a single 'universal' data-generating process for elections. However, the closer similarities among the results for the first four processes (Politbarometer, ANES, multi-dimensional spatial, and – particularly where resistance to strategy is concerned – IAC) than with the remaining two (one-dimensional spatial, and IC) suggest that the former differ significantly from the latter in their similarity to actual elections.

Indeed there are good reasons to expect the last two processes to possess less verisimilitude than the first four. We noted above in section 3.2 that IC is unrealistic, in predicting closer elections than are actually observed. The one-dimensional spatial model implies that in any election, all but two candidates will never be the last choice of any voter. This is reasonable enough if the candidates are policy options in a linear space (such as how much to spend on a particular project), but it is totally inconsistent with ranking data that we observe in multi-candidate elections for human office-holders.

9.2. Resistance to strategy

With regard to resistance to strategy (R) scores, the first result we note is that no compound 'Condorcet-X' voting rule ever has a lower score than its corresponding voting rule 'X'. This is consistent with our proposition 2 above, which pertains to the voting rules that possess CMD. Further, it suggests that the circumstances under which a Condorcet provision introduces the possibility of manipulation to a base rule not possessing CMD are relatively rare.

Second, we note that the Condorcet-elimination rules with $p \leq 1/2$ always have the same R scores as their corresponding elimination rules; this is consistent with our proposition 3.

Third, we note that the four consonant data-generating processes produce manipulability results that are strikingly similar, as seen in figure 2. They agree that Hare and Condorcet-Hare have the greatest resistance to strategy, that the R scores of elimination rules decrease monotonically with p, that Condorcet-elimination rules improve on the R scores of their corresponding elimination rules to an increasing degree as p increases above $1/2$, that the R scores of positional rules approximate a clockwise-tilted U shape in p, and that the R scores of Condorcet-positional rules approximate a less-tilted U shape in p. The results from the one-dimensional spatial model are anomalous in that Condorcet-anti-plurality has a higher R score than Hare and Condorcet-Hare, and in the changed shapes of the elimination and

Condorcet-elimination curves. The results from the IC model are anomalous in that in many rules have R scores close to 0; the major exceptions are the elimination and Condorcet-elimination rules with p close to 0, and the positional and Condorcet-positional rules with p close to 1.[42]

9.3. Utilitarian efficiency

With regard to utilitarian efficiency (E) scores, all six data-generating processes agree that the positional continuum varies most with p, forming an inverted U shape. According to the Politbarometer, ANES, multi-dimensional spatial, and IC processes, the elimination continuum also forms an inverted U shape in p, but one that is far flatter. The Politbarometer, ANES, and both spatial processes agree that the Condorcet-consistent continua vary less with p than both of the other continua. (In the case of the one-dimensional spatial model, neither varies at all with p, or ever differs from the other, because there is always a Condorcet winner.) Using five of the six data-generating processes, a positional rule achieves the highest E score among ranked ballot rules (although, when the ANES data are used, this distinction is shared in a three-way tie among a positional rule, an elimination rule and a Condorcet positional rule). However, the one-dimensional spatial model results in all positional rules having strictly lower E scores than the Condorcet-consistent rules.[43]

9.4. Comparison to the literature

In addition to having a high degree of internal consistency across data-generating processes, our results are broadly consistent with the existing literatures on resistance to strategy and utilitarian efficiency. For example, papers in the first literature commonly find that Hare has a high resistance to strategy (in the sense of having a relatively low probability of being manipulable), while Borda has a low resistance to strategy and plurality has an intermediate resistance to strategy.[44] Papers in the second literature commonly find that Borda and approval have relatively high utilitarian efficiency, while plurality has relatively low utilitarian efficiency.[45] We extend both of these literatures by considering an unusually comprehensive range of both voting rules and data-generating processes. For example, few authors have measured the resistance to strategy or utilitarian efficiency of Condorcet-Hare.[46] We also extend the literatures by measuring both of these attributes in combination, and illuminating the tradeoffs that can be made between these two criteria by the choice of a voting rule.

[42] It is worth noting that the IC model with smaller numbers of voters leads to R scores that correlate more closely to the other data-generating processes. For example, with 9 voters, the IC model has an average correlation of 92.4% with the four most consonant data-generating processes. However, we present the results with 99 voters here with the idea that it would be inconsistent to suddenly shift our focus to such small elections.

[43] This result also holds when we measure utilitarian efficiency as the average sum of utilities from the winning candidate, rather than the share of trials in which the candidate who maximizes the sum of utilities is chosen. This shows that theorem 3.1 from Apesteguia et al (2011) does not extend to all data-generating processes.

[44] For example, Chamberlin (1985), Lepelley and Valognes (2003), Favardin and Lepelley (2006), Tideman (2006), and Green-Armytage (2014) all agree on these points.

[45] For example, Weber (1978), Bordley (1983), and Merrill (1984) all agree on these points.

[46] The only examples that we are aware of are Tideman (2006) and Green-Armytage (2011), which provide measures of resistance to strategy, but not utilitarian efficiency.

For most of our data-generating processes, all but the very smallest of differences between scores that we calculate for voting rules are statistically significant. The 95% confidence interval around one of our estimated R (or E) scores is on the order of $\pm 1.96\sqrt{R(1-R)/T}$, where T is the number of synthetic elections used. When T = 126,637 (as in our Politbarometer process), the upper bound of this (i.e. when $R = 1/2$) is approximately 0.0028, when T = 847 (as in our ANES process), the upper bound is 0.0337, and when T = 30,000 (as in our other processes), the upper bound is 0.0057. Furthermore, the confidence interval around the estimated difference between the scores of any two rules is often far smaller than this. That is, let Ξ_i be equal to 1 if rule X succeeds in trial i, and equal to 0 otherwise. Let Ψ_i be defined analogously for rule Y. The variance of the difference between the scores of rule X and rule Y is equal to the variance of Ξ, plus the variance of Ψ, minus twice the covariance of Ξ and Ψ. These terms often largely cancel, leading to even smaller variances and confidence intervals than what our first approximation suggests.

10. The efficiency-resistance frontier

Return to the idea of a frontier of voting rules such that a higher E score can only be obtained at the cost of a lower R score. Begin with random dictatorship, which has R and E scores of 1 and 0.5181, respectively, according to our Politbarometer results. Next on the frontier is Condorcet-Hare, which has R and E scores of 0.9804 and 0.9516, respectively, which implies that an increase in E of 0.4335 is obtained at a cost of a decrease in R of 0.0196; this is quite a large gain in exchange for a relatively small loss.

However, tradeoffs along the frontier beyond this point are substantially less favorable. Relative to Condorcet-Hare, Condorcet-anti-plurality gains only 0.0008 in E in exchange for a loss of 0.1198 in R. Approval relative to Condorcet-Hare gains 0.0021 in E at a cost of losing 0.2616 in R. Normalized range gains 0.0034 in E at a cost of 0.2775 in R. Range voting gains 0.0484 in E at a cost of losing 0.7990 in R. Other rules, such as Baldwin, Black, and minimax, lie inside the frontier.

The ANES data and multi-dimensional spatial models produce frontiers with very similar shapes in ER space: a gentle slope from random dictator to Condorcet-Hare, and then a steep cliff with other rules on the face of the cliff and range at its base. The IAC model results in a cliff that is somewhat less steep, but which still has an average slope of approximately -4.89 from Condorcet-Hare to Borda, and an average slope of -6.92 from Condorcet-Hare to Black.

Thus, in situations where honesty can be assumed (for example, perhaps a situation in which close friends are voting among alternative restaurant plans), the superior utilitarian efficiency of range voting might make it the most attractive rule. Conversely, in situations where avoiding strategic voting is the greatest concern, the random dictator rule may be seen as a logical choice. However, in situations where both criteria are important, the Condorcet-Hare rule, among the rules we consider, appears to offer the most attractive combination of resistance to strategy and utilitarian efficiency.

11. Conclusion

Two foundational political scholars, Condorcet and Hare, worked independently in different countries and centuries, and described two procedures for analyzing ballots: the method of pairwise comparison, and the method of iteratively eliminating the plurality loser. We find that these two procedures have strongly complementary properties when combined into a single voting rule, i.e. a Condorcet-Hare rule. Our results suggest that academics and reformers should give more attention to Condorcet-Hare rules as practical alternatives to other single-winner election systems.

Borda, a contemporary of Condorcet's, described a procedure that scores well in utilitarian efficiency but poorly in resistance to strategy. Thus, it is very likely to elect the candidate who maximizes the sum of utilities when votes are sincere, but it is also likely to provide incentives for strategic behavior, which makes it more difficult to guess how much efficiency it should be expected to provide in practice.

Meanwhile, much of the world's population continues to use the plurality rule. While this has the advantage of simplicity, we find that it is clearly dominated in both dimensions by a substantial number of other voting rules. This finding adds another clear argument to the case for voting reform.

Appendix: Proofs of propositions 1-3

Proof of proposition 1, part 1: *Positional rules with $p > 1/2$ lack CMD.*

Here we show that for any $p > 1/2$, there is an election that a specified majority cannot win. Given a value of $p > 1/2$, let n be an integer greater than $(2-p)/(2p-1)$.

Consider an election in which there are $2n$ voters in the minority and $2n+2$ voters in the majority. Specify further that those in the minority have cast n votes for B $\succ$ C $\succ$ A and n votes for $C \succ B \succ A$. We have assumed that every vote must list a second choice as well as a first choice. Thus those who want A to win must give p points to either B or C for each point that they give to A. If it is possible at all for A to win, the hurdle to be overcome will be lowest if the A voters give the same number of points to B and C, so that B and C will have the same number of points after the election. This can be accomplished by casting $n+1$ votes for A $\succ$ B $\succ$ C and $n+1$ votes for $A \succ C \succ B$. Then the result of the election is that A has $2n+2$ points, whereas B and C each have $n+2np+p$ points. Thus A wins if and only if

$$n + 2np + p < 2n + 2$$
$$n < (2-p)/(2p-1)$$

But n has been assumed to be greater than this. Thus the specified majority cannot elect A.

Proof of proposition 1, part 2: *Positional rules with $p \le 1/2$ possess CMD.*

Assume that the majority want A. Let the majority appoint one of their own voters to counter each voter who ranks B or C first, in the following manner: The counter-voter ranks A first followed by B and C in the opposite of the order used by the voter being countered. After such countering, no pair (voter plus counter) adds more to B or C than to A. (For example, if a minority voter ranks B $\succ$ C $\succ$ A and is countered with A $\succ$ C $\succ$ B, then the pair add 1, $2p$, and 1 to the scores of A, B, and C respectively. If a minority voter ranks B $\succ$ A $\succ$ C and is countered with A $\succ$ C $\succ$ B, the pair adds $1+p$, 1, and p.) Majority voters who are not needed for countering rank A first, and the remaining candidates in either order. Thus the majority can achieve the election of A no matter how the minority votes. ■

Proof of proposition 2:

(1) Let A be the winner when voting rule X is applied to preference profile Π. Since sincere voting is a core equilibrium, there is no candidate B ≠ A such that those who prefer B to A can change their votes so as to make B the winner under voting rule X.

(2) Since, by assumption, voting rule X possesses CMD, point 1 implies that there is no candidate B ≠ A such that a majority of voters prefer B to A. (That is, if a majority preferred B to A, this majority would be able to make B the winner, since voting rule X possesses CMD.)

(3) Point 2 implies that no candidate other than A could possibly be a sincere Condorcet winner for profile Π. Therefore, since A is the only candidate who could be a Condorcet winner for profile Π, A must be the sincere winner for Condorcet-X as well as X.

(4) Point 2 implies that there is no candidate B ≠ A such that those who prefer B to A can change their votes so as to make B a Condorcet winner. That is, from point 2, there is not a majority of voters in favor of B over A, and since those who prefer B to A already rank B above A (because Π is a profile of sincere preferences), there is nothing that those who prefer B to A can do to give B a majority over A.

(5) Combining points 1, 3, and 4: A is the sincere Condorcet-X winner, and for any candidate B ≠ A, voters preferring B to A cannot change their votes either so as to make B the winner according to the voting rule X, or so as to make B the Condorcet winner. Therefore, they cannot make B the winner according to the voting rule Condorcet-X. ■

Proof of proposition 3, case 1: *Candidate A is a sincere Condorcet winner.*

Case 1-1: A is the sincere elimination-p winner as well as a sincere Condorcet winner. In this case, we want to show that if elimination-p is manipulable, Condorcet-elimination-p is also manipulable. So, suppose that the voters who prefer B to A can make B the elimination-p winner. This implies that they can vote in such a way as to create a situation in which A is the positional-p loser, and B beats C pairwise. The question is whether this necessarily implies that they can arrange for C to beat or tie A pairwise at the same time, so that there is no Condorcet winner and thus the elimination-p rule decides the Condorcet-elimination-p winner.

To address this question, define a, b, and c as the numbers of sincere first-choice votes for A, B, and C, respectively; and define $V \equiv a + b + c$. To provide the most critical test, we want to explore the case in which it is *easiest* for strategizers to make A the positional-p loser *without* being able to make C pairwise-defeat A, so we suppose that there are no voters with the sincere ordering $\mathrm{C} \succ \mathrm{A} \succ \mathrm{B}$, and that the strategizers are able to vote so that first-round points are evenly divided between B and C (that is, that they can use their own ballots to balance out the difference in points between B and C from the ballots of non-strategizers). Thus, in the strategic voting situation that we are examining, the A voters' ballots give a points to A, and ap points that are divided between B and C, while the B and C voters' ballots give $b + bp + c + cp$ points that are divided between B and C. Since A is to be made the positional-p loser, we know that

$$a < \frac{1}{2}(b + c + ap + bp + cp)$$

$$a < \frac{1}{3}(1 + p)V$$

In this situation, we know that with the strategic votes, A pairwise-defeats B, B pairwise-defeats C, and A is the positional-p loser. To guarantee that B is the Condorcet-elimination-p winner, all that remains is to show that strategizers can cause C to pairwise-defeat (or pairwise-tie) A at the same time, causing a cycle that will be resolved in B's favor. Since all voters other than those whose sincere first choice is A rank C above A in the situation that is optimal for the strategizers, we can represent this requirement by $a \leq V/2$. From the expression above, this is true if $(1/3)(1+p)V \leq V/2$, and thus it is true if $p \leq 1/2$.

Case 1-2: While A is a Condorcet winner, another candidate, B, is the sincere elimination-p winner. In this case, the elimination-p rule is certainly manipulable; the majority who prefer A to B just need to rank A first and allocate their second choices so that neither B nor C has a higher positional score than A. The algebra employed in case 1-1 can be used again to show that if $p \leq 1/2$, those who prefer B to A can make B the Condorcet-X winner.

Proof of proposition 3, case 2: *There is no sincere Condorcet winner.* Let A be the sincere elimination-p winner.

Case 2-1: There is another candidate B who is preferred by a majority to A. In this case, both elimination-p and Condorcet-elimination-p must be manipulable, because both possess CMD.

Case 2-2: There is not another candidate B who is preferred by a majority to A. (That is, candidate A has a pairwise tie with one or both other candidates.) A is the sincere winner in both rules, and the algebra employed in case 1-1 can be used again to show that if $p \leq 1/2$, the conditions necessary for those who prefer B to make B the elimination-p winner are also sufficient to ensure that they can make B the Condorcet-elimination-p winner. ■

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